Published by Institute of Physics Publishing for SISSA

Received: *February 11, 2008*
Revised: *March 16, 2008*
Accepted: *June 6, 2008*
Published: *June 19, 2008*

JHEP06(2008)067

Macroscopic screening of $1/r$ potentials from UV/IR-mixing

Robert C. Helling[ab] **and Jiangyang You**[b]

[a] *Arnold Sommerfeld Center, Ludwig-Maximilians-Universität München, Theresienstraße 37, 80333 München, Germany*
[b] *School of Engineering and Science, Jacobs University, Postfach 750561, 28725 Bremen, Germany*
E-mail: helling@atdotde.de, j.you@iu-bremen.de

Abstract: We compute the static potential in a scalar field theory in non-commutative space including a term due to UV/IR-mixing. As a result, the potential decays exponentially fast with distance rather than like a power law as in the commutative theory due to the exchange of massless particles. This shows that when quantum effects are taken into account the introduction of non-commutativity not only modifies physics at short distances but has dramatic macroscopic consequences as well. As a result, we give a *lower* bound on the scale of non-commutativity (if present at all) to be compatible with power law forces over macroscopic distances.

Keywords: Field Theories in Higher Dimensions, Non-Commutative Geometry.

Contents

1. Introduction

We have two extremely successful approaches to understand the fundamental interactions: Quantum field theory exemplified in the standard model of particle physics and general relativity governing the gravitational forces dominating the universe at large scales. It is widely believed that for a unification of these two approaches one has to develop a description of a quantum space-time which has to approximate the structure of a differentiable manifold at distances larger than the Planck scale.

Probably the most conservative approach to such a quantum space-time is to promote the coordinates to quantum operators which no longer commute and thereby induce a fundamental quantum uncertainty of geometry at the Planck scale [1, 2] . This scenario gained a lot of momentum a few years ago when it became clear that it arises naturally from string theory in backgrounds with non-vanishing Kalb-Ramond 2-form $B_{\mu\nu}$ [3 – 5] .

The simplest version of space-time non-commutativity is obtained from commutation relations

$$[x^\mu, x^\nu] = i\theta^{\mu\nu}$$

for a constant, anti-symmetric matrix θ. It leads to the Moyal-Weyl *-product for fields living in this space time

$$(f * g)(x) = e^{\frac{i}{2}\theta^{\mu\nu}\frac{\partial}{\partial y^\mu}\frac{\partial}{\partial z^\nu}} f(y)g(z)\Big|_{x=y=z}$$

By replacing the ordinary commutative product between fields in the action many field theories including U(N) gauge theories can be adapted to this non-commutative space-time. It was argued for example in [6] that by treating θ perturbatively one could even deform the standard model with differences showing up only at microscopic scales. Due to UV/IR-mixing, this perturbative treatment is not justified and in this paper we explicitly work out macroscopic consequences of non-commutativity due to UV/IR-mixing.

In simple non-commutative theories, θ plays the role of a non-commutative deformation parameter as does $\hbar$ in quantum mechanics where it controls the extend to which observables cease to commute. Then, the correspondence principle states that the quantum deformation of the classical theory is only noticeable at small scales where variations of the classical action are of the order of $\hbar$. Similarly, one could hope that a space-time non-commutativity with the entries of θ being of the order of ℓ_P^2 would only be relevant for physics at Planckian distances or with typical momenta of the order of the Planck mass while macroscopic physics would be indistinguishable from a theory on a commutative space.

This expectation was shaken by [7] where loop corrections to non-commutative scalar field theory were computed. There it was found that while planar diagrams receive no modifications due to non-commutativity, loop momentum integrals of non-planar diagrams have additional phase factors which generically render integrals which are UV divergent in the commutative theory finite. Thus the non-commutativity acts as a UV regulator.

For special external momenta, and most importantly $p \to 0$, the regulator is not effective and the UV divergence reappears now in the form of a IR divergence. Quadratic divergences of the commutative theory thus lead to terms of the form

$$\phi \frac{1}{p \circ p} \phi = \phi \frac{1}{p^\mu \theta_{\mu\nu} \theta^{\nu\rho} p_\rho} \phi \tag{1.1}$$

in the effective action. These terms notably are not of the form of the *-product interactions of the classical theory and it seems likely that at each loop order new counter-terms are needed. This would render the non-commutative theories non-renormalisable.

This expectation turned out to be too pessimistic as in [8, 9] it was shown that ϕ^4 theory on non-commutative $\mathbb{R}^4$ is in fact renormalisable. There it was shown that only a single further counter-term is required. In those papers, for technical reasons this counter-term was taken to be

$$x^\mu * \phi * \phi * x_\mu \tag{1.2}$$

to make the action symmetric under Fourier transform. It cancels the divergence of the form (1.1) as it has the same scaling dimension and $\theta_{\nu\mu} x^\nu *$ acts like ∂_μ. Therefore it was suggested [10] that also a term of the form (1.1) leads to a renormalisable theory. This renormalisability has in the meantime been firmly established in [11]. This motivates the approach we will take in this paper: The term (1.1) is not just the result of a one loop calculation but it should be thought of as part of the quadratic part of the effective action which if not present right from the start is generated by quantum corrections. Its contribution can be resummed by instead of the usual Feynman propagator using the functional inverse of $\Box + \frac{1}{p \circ p}$ as the propagator.

Thus we take our theory to be given by

$$\mathcal{S} = \int d^4x \left(\partial^\mu \phi \partial_\mu \phi + \phi \frac{1}{p \circ p} \phi - \frac{\lambda}{4!} \phi * \phi * \phi * \phi \right) \tag{1.3}$$

where as usual $p_\mu = i\partial_\mu$. Note well that this new term is non-perturbative in θ and becomes singular in the commutative limit. The ϕ^4 interaction leads to the well known 1-loop amplitudes that cause UV/IR-mixing and a counter-term (1.1). Once this resulting counter-term has been taken into account, the ϕ^4-interaction will not play a further role

JHEP06(2008)067

in this paper computing the static potential which follows from the quadratic part of the effective action.

In addition, one could consider a mass term but as long as the mass is smaller than $1/\sqrt{\theta}$ it will not change our qualitative findings, an exponentially decaying static potential with characteristic length $\sqrt{\theta}$ and thus — for clearness of presentation — we will not consider it. If present, a mass term would lead to a Yukawa type effective potential. Thus, if the interaction has decayed exponentially due to the mass already over length scales smaller than the scale of non-commutativity, i.e. $1/m \ll \sqrt{\theta}$, the effect of non-commutativity will be masked. We will not further consider this case.

In the two remaining terms, the only apperance of θ is in the counter-term. Thus, by rescaling θ, we can arrange the numerical coefficient of the counter-term to unity and we do not need a further numerical coefficient.

The choice of considering a scalar field is merely for clarity of presentation. We expect the discussion of this paper to generalize to any other field theory with UV/IR-mixing. For example, the authors of [12] found the self-energy correction for the photon to be $\Pi_{\mu\nu} \sim (\theta p)_\mu (\theta p)_\nu/(p \circ p)^2$ which has the same scaling with as the term considered in this paper. Therefore, we expect it to lead to qualitatively similar results. This should justify our misnomer "Coulomb potential" for a $1/r$ potential transmitted by a massless scalar instead of a vector field.

In the existing literature, an effective action including this one loop induced term has been discussed as giving a potential in momentum space which has its minimum away from $p_\mu = 0$ thus leading to a spontaneous breaking of translation invariance and the formation of stripes [13 – 16]. Lattice calculations have confirmed that this effect persists non-perturbatively [17, 18] (see [19, 20] in the case of the fuzzy sphere).

In this paper however, we take a different point of view and treat all terms quadratic in the field as part of the free theory determining the propagator. We will find that instead of the power law behavior of the potential as $1/r^{D-3}$ in the commutative theory the non-commutative potential is decaying exponentially fast. Consequently, the force the field is transmitting is short ranged and effectively cut-off at the scale of the non-commutativity. We find that even a small amount of non-commutativity has drastic effects at macroscopic scales as it screens long range forces. Our conclusion will thus be that it is not correct to think of non-commutativity modifying a theory only at short scales — possibly the Planck scale. As soon as quantum effects which result in UV/IR mixing are considered, the theory is drastically changed at scales large compared to the non-commutativity.

2. Screened Coulomb interaction

For concreteness, let us work in four-dimensional space. We want to compute the potential due to a source

$$J(x) = \int d\tau \, \delta^{(4)}(x - r(\tau)).$$

Let us assume the source is static and the force is mediated by a field with momentum space propagator $G(p) = G(E, \vec{p})$. Then the potential is given by

$$V(t, \vec{x}) = \int d^4x' \int d^4p\, G(p) e^{ip(x-x')} J(x') = \int d^3\vec{p}\, G(0, \vec{p}) e^{i\vec{p}\cdot(\vec{x}-\vec{r})}.$$

Thus, in order to obtain the potential we have to compute the spatial Fourier transform of the propagator at $E = 0$.

For the scalar theory with the action (1.3), the momentum space propagator is given by

$$G(p) = \frac{1}{p^2 + \frac{1}{p \circ p}}.$$

As a warm up, we will consider a Euclidean theory with self-dual

$$\theta^{\mu\nu} = \begin{pmatrix} 0 & \theta & 0 & 0 \\ -\theta & 0 & 0 & 0 \\ 0 & 0 & 0 & \theta \\ 0 & 0 & -\theta & 0 \end{pmatrix},$$

as in this case $p \circ p = \theta^2 p^2$ and the propagator only depends on p^2. We can easily evaluate the static potential in polar coordinates:

$$V(r) = \int d^3\vec{p}\, \frac{e^{i\vec{p}\cdot\vec{r}}}{\vec{p}^2 + \frac{1}{\theta^2 \vec{p}^2}} = 2\pi^2 \frac{e^{-\frac{r}{\sqrt{2|\theta|}}} \cos\left(r/\sqrt{2|\theta|}\right)}{r} \qquad (2.1)$$

The effect of the non-commutativity is to dress the $1/r$ Coulomb potential with an oscillatory factor and an exponential decay over the scale of the non-commutativity. The power law character of the force law has been replaced by a finite interaction range governed by the length scale of the non-commutativity. Indeed, the deviation from Coulomb behavior is most pronounced at macroscopic distances $r \gg \sqrt{|\theta|}$ and we have to conclude that it is incorrect to think of the non-commutativity as a correction affecting physics only at very short distances.

Form a deformation perspective, it might come as a surprise that the commutative $1/r$ potential is not recovered as a smooth $\theta \to 0$ limit but in the opposite $\theta \to \infty$ limit. This however is due to the well known nature of the UV/IR-mixing term which blows up for small θ. Although this has been clear since the discovery of UV/IR-mixing the continuity of the commutative limit is still tacitly assumed in approaches using an expansion in θ as used for example in applications of the Seiberg-Witten-map. Obviously, this expansion is only valid at the classical level and breaks down at the quantum level.

Our result demonstrates that it is essential to fully resum the contribution of all orders in θ in order not to miss the effects of the non-locality of the *-product.

Let us consider a more general non-commutativity. Up to now, we have used euclidean signature and the self-dual choice of $\theta^{\mu\nu}$ implied that both p^2 and $p \circ p$ were SO(4) symmetric. In Lorentzian signature, it is well known that having non-commutativity between time and space can lead to problems with unitarity unless it is possible to boost to a frame

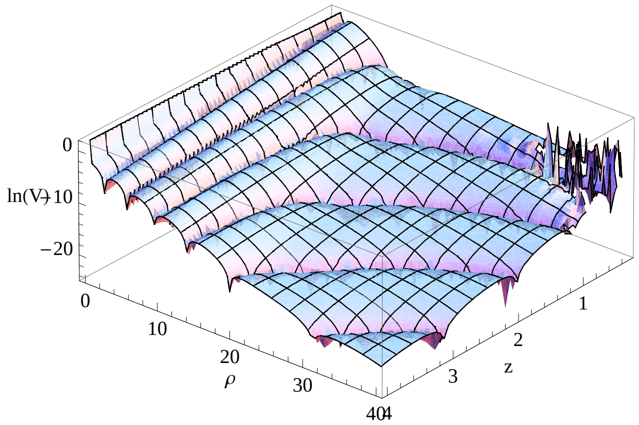

Figure 1: The logarithm of the static potential, coordinates in units of $\sqrt{\theta}$. The exponential decay of the potential for $\rho, z \to \infty$ can be seen. There is some numerical noise at small z.

where time is a commuting coordinate. We will now assume that this boost has been performed. Then, up to a rotation, we can assume $\theta^{\mu\nu}$ is of the form

$$\theta^{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \theta & 0 \\ 0 & -\theta & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

The non-commutativity is only in the xy-plane and $\theta > 0$. In cylindrical coordinates, after a change of coordinates $p \mapsto p/\sqrt{\theta}$, we have to evaluate

$$V(\varrho, z) = \frac{1}{\sqrt{\theta}} \int_0^\infty p_\varrho \, dp_\varrho \int_0^{2\pi} dp_\phi \int_{-\infty}^{\infty} dp_z \, \frac{e^{i(p_\varrho \varrho \cos p_\phi + p_z z)/\sqrt{\theta}}}{p_z^2 + p_\varrho^2 + \frac{1}{p_\varrho^2}}$$

Figure 1 shows this static potential evaluated numerically. One can see it is indeed decaying exponentially fast.

The p_ϕ and p_z integrals are easily evaluated and we find

$$V(\varrho, z) = \frac{4\pi^2}{\sqrt{\theta}} \int_0^\infty dp_\varrho \, p_\varrho \frac{e^{-|z|\sqrt{p_\varrho^2 + \frac{1}{p_\varrho^2}}/\sqrt{\theta}} J_0(p_\varrho \varrho/\sqrt{\theta})}{\sqrt{p_\varrho^2 + \frac{1}{p_\varrho^2}}}.$$

We cannot evaluate this remaining integral analytically. However, it is sufficient to determine it asymptotically for distances $\sqrt{\varrho^2 + z^2}$ large compared to $\sqrt{\theta}$ using the method of steepest descent. We fix the direction $c = \varrho/|z|$ and use the asymptotic form of the Bessel function

$$J_0(x) \to \sqrt{\frac{2}{\pi x}} \cos(x - \pi/4).$$

Eventually

$$V(cz, z) \to \sqrt{\frac{32\pi^3}{\sqrt{\theta} c|z|}} \int_0^\infty dp_\varrho \sqrt{\frac{p_\varrho}{p_\varrho^2 + \frac{1}{p_\varrho^2}}} e^{-|z|\sqrt{p_\varrho^2 + \frac{1}{p_\varrho^2}}/\sqrt{\theta}} \cos\left(\frac{p_\varrho c|z|}{\sqrt{\theta}} - \frac{\pi}{4}\right). \tag{2.2}$$

JHEP06(2008)067

Next, we rewrite the cosine as a sum of two complex exponentials. The integral contains $|z|$ only in form of the exponential $\exp(\Phi(p_\varrho)|z|/\sqrt{\theta})$. Thus, for $|z| \gg \sqrt{\theta}$ it is dominated by the contribution from the points where the "complex phase" $\Phi(p_\varrho) = \sqrt{p_\varrho^2 + \frac{1}{p_\varrho^2}} + ic\alpha p_\varrho$ is stationary (with $\alpha = \pm 1$). For each α there are contributions from four stationary points $p_{\varrho,i}$ (which are a function of c only but not of z or θ) obeying

$$p_{\varrho,i} - \frac{1}{p_{\varrho,i}^3} = -i\alpha c\sqrt{p_{\varrho,i}^2 + \frac{1}{p_{\varrho,i}^2}}. \tag{2.3}$$

By evaluating the p_ϱ integral to leading order in $z/\sqrt{\theta}$ we find

$$V(z, cz) = \sum_{\pm,i} \frac{4\pi^2}{|z|}\left(1 + \frac{1}{p_{\varrho,i}^4}\right)^{\frac{1}{4}} \frac{\pm p_{\varrho,i}^4}{\sqrt{c(2 + 6p_{\varrho,i}^4)}} e^{\pm(\Phi(p_{\varrho,i})|z|/\sqrt{\theta} - \pi i/4)}\left(1 + O\left(\frac{\sqrt{\theta}}{|z|}\right)\right) \tag{2.4}$$

as explained in the appendix. As detailed there, the sum has to be taken over all solutions $p_{\varrho,i}$ of

$$\frac{1}{p_{\varrho,i}^4} = 1 - \frac{c^2 \pm c\sqrt{c^2 - 8}}{2}. \tag{2.5}$$

By analyzing the integrand of (2.2) for large and small p_ϱ it can be seen that $V(cz, z) \to 0$ for large z. Thus we can conclude that the sign in the exponential in (2.4) will always be such that the real part of the coefficient of $|z|$ is positive. Thus, the potential is in fact decaying rather than growing exponentially. In the appendix, it is shown algebraically that Φ is indeed stationary only for points p_ϱ such that the real part of $-\Phi(p_\varrho)|z|/\sqrt{\theta}$ is negative.

The expression (2.4) constitutes the main result of this work. It shows that for distances z large compared to the length scale of non-commutativity $\sqrt{\theta}$, the interaction which is transmitted by the field ϕ dies off exponentially fast in contrast to the commutative situation where it only falls off like $1/|z|$. The effect is as if due to the non-commutativity the field ϕ had acquired a direction dependent mass of the order of $1/\sqrt{\theta}$.

3. Conclusions

In this paper, we investigated the effect of non-commutativity on the long range interactions mediated by a scalar field taking into account UV/IR mixing. This effect due to loop diagrams induces new quadratic terms in the effective action with negative powers of θ, the scale of non-commutativity.

We showed that by including these terms into a resummed propagator the power law effective potential is modified to a Yukawa type potential with exponential fall-off with a direction dependent decay length of the order of $\sqrt{\theta}$.

This calculation shows explicitly that the assumption motivated by a classical analysis that the deformation leading to non-commutativity of space-time coordinates only modifies physics at short distances (typically the Planck scale) is not true once quantum effects are

JHEP06(2008)067

taken into account. Rather, the drastic consequences of the non-commutativity is that it switches off the interaction for macroscopic scales.

This raises the question of viability of simple non-commutative models for phenomenology given that at macroscopic scales we do observe long range power-law interactions but no signs of non-commutativity. In this paper we have only treated a single scalar field but the argument should generalize to fields of higher spin as soon as the theory has UV/IR-mixing. The case of gauge fields is known [12] to have a similar self-energy correction proportional to $(\theta p)^{-2}$ although with a different tensor structure. We expect this to lead to qualitatively similar effects.

The specific term $1/p \circ p$ arises from integrals which are quadratically divergent in the commutative theory. Thus the argument should directly apply for example for the Higgs self-energy in the standard model giving a lower bound on $\sqrt{\theta}$ at the order of the inverse Higgs mass. Terms with milder divergence lead to slightly different terms in the effective action which however will have a similar behavior that they become large for small $\sqrt{\theta}$ since they arise from divergences which are regularized by the non-commutativity. Thus they are likely to lead to similar macroscopic consequences of the non-commutativity and to result in similar bounds.

We think this is an example of the generic situation that quantum effects transmit non-localities (here due to non-commutativity) which classically appear only at short scales to arbitrarily large scales. This should be seen as a challenge to model-building based on non-commutative space-time or other sources of non-locality.

Acknowledgments

We have profited a lot from discussions with Harald Grosse and Peter Schupp. Both of us are grateful to Jacobs University for financial support. RCH thanks Elitenetzwerk Bayern for funding.

A. The effective potential is decaying exponentially

In this appendix, it will be shown that the real part of the phase $\Phi(p_{\varrho,i})$ at the stationary points is positive and thus the effective potential in (2.4) is indeed decaying at large distances.

We want to determine the sign of the real part of

$$\Phi(p_{\varrho,i}) = \sqrt{p_{\varrho,i}^2 + \frac{1}{p_{\varrho,i}^2}} + i\alpha c p_{\varrho,i}$$

at its stationary points with $\alpha = \pm 1$ and $c \geq 0$. As we pick the branch-cut to be along the negative real axis, all square-roots will have non-negative real parts. Thus it is sufficient to show that at the critical points we have $\alpha \, \mathrm{Im}(p_{\varrho,i}) \leq 0$.

Setting the derivative of Φ to zero yields

$$\alpha \left(p_{\varrho,i} - \frac{1}{p_{\varrho,i}^3} \right) = -ic\sqrt{p_{\varrho,i}^2 + \frac{1}{p_{\varrho,i}^2}}. \tag{A.1}$$

JHEP06(2008)067

Squaring this relation gives a quadratic equation for $1/p_{\varrho,i}^4$ which is solved by

$$\frac{1}{p_{\varrho,i}^4} = 1 - \frac{c}{2}\left(c + \alpha\beta\sqrt{c^2 - 8}\right) \tag{A.2}$$

with another sign $\beta = \pm 1$. As to obtain this solution we have squared (A.1), only four of the eight solutions for $p_{\varrho,i}$ in (A.2) will really solve (A.1). However, the other solutions will solve (A.1) for the other choice of α. Therefore, instead of fixing α and determining which four of the eight $p_{\varrho,i}$ is really a solution, we find the appropriate α_i for each of the four $p_{\varrho,i}$.

Let us first consider the case $c^2 \geq 8$. From (A.1) and the non-negative real part of square-roots we find that the imaginary part of $\alpha_i p_{\varrho,i}(1 - 1/p_{\varrho,i}^4)$ is negative. But (A.2) implies that $1 - 1/p_{\varrho,i}^4$ is real and positive. Thus the imaginary part of $\alpha_i p_{\varrho,i}$ is negative and indeed $\exp(-|z|\Phi(p_{\varrho,i})/\sqrt{\theta})$ decays for large $|z|$.

It remains to consider $c^2 < 8$. As with any solution $p_{\varrho,i}$ of (A.2) also $i^n p_{\varrho,i}$ are solutions, we can first consider the solution such that $|\arg(p_{\varrho,1})| \leq \pi/4$. In this case, it is easy to see that the imaginary part of $p_{\varrho,1} - 1/p_{\varrho,1}^3$ has the same sign as the imaginary part of $p_{\varrho,1}$. Again using (A.1) and the non-negative real part of square-roots implies that $\mathrm{Im}(\alpha_1 p_{\varrho,1})$ is not positive. Substituting the next solution $p_{\varrho,2} = -p_{\varrho,1}$ in (A.1), the r.h.s. does not change and we have $\alpha_2 = -\alpha_1$ to compensate the change of sign of $p_{\varrho,i} + 1/p_{\varrho,i}^3$. Thus $\alpha_1 p_{\varrho,1} = \alpha_2 p_{\varrho,2}$.

Next, consider $p_{\varrho,3} = i p_{\varrho,1}$ which has positive imaginary part. For any complex number $z = x + iy$ one has

$$\mathrm{Im}\left(z^2 + \frac{1}{z^2}\right) = 2xy\left(1 - \frac{1}{|z|^4}\right)$$

From the explicit solution we have $|1/p_{\varrho,i}^4| = 1 + c^2$. Thus $\mathrm{Im}(p_{\varrho,1}^2 + 1/p_{\varrho,1}^2)$ has the opposite sign of $\mathrm{Im}(p_{\varrho,1})$. Using this together with $\sqrt{-z} = -i\sqrt{z}\,\mathrm{sgn}(\mathrm{Im}(z))$ in evaluating the r.h.s. of (A.1) for $p_{\varrho,3}$ gives $\alpha_3 = -1$. Thus $\mathrm{Im}(\alpha_3 p_{\varrho,3}) \leq 0$ as well. The same is true for $p_{\varrho,4} = -p_{\varrho,3}$ as can be shown using the same argument as in the case of $p_{\varrho,2}$.

This concludes our proof that for all stationary points $p_{\varrho,i}$ of Φ the sign α_i adjusts to make $\mathrm{Im}(\alpha_i p_{\varrho,i}) \leq 0$ and thus the effective potential decays at large distances.

References

[1] H.S. Snyder, *Quantized space-time*, *Phys. Rev.* **71** (1947) 38.

[2] S. Doplicher, K. Fredenhagen and J.E. Roberts, *The quantum structure of space-time at the Planck scale and quantum fields*, *Commun. Math. Phys.* **172** (1995) 187 [hep-th/0303037].

[3] M.R. Douglas and C.M. Hull, *D-branes and the noncommutative torus*, *JHEP* **02** (1998) 008 [hep-th/9711165].

[4] N. Seiberg and E. Witten, *String theory and noncommutative geometry*, *JHEP* **09** (1999) 032 [hep-th/9908142].

[5] M.R. Douglas and N.A. Nekrasov, *Noncommutative field theory*, *Rev. Mod. Phys.* **73** (2001) 977 [hep-th/0106048].

JHEP06(2008)067

[6] X. Calmet, B. Jurčo, P. Schupp, J. Wess and M. Wohlgenannt, *The standard model on non-commutative space-time*, *Eur. Phys. J.* **C 23** (2002) 363 [hep-ph/0111115].

[7] S. Minwalla, M. Van Raamsdonk and N. Seiberg, *Noncommutative perturbative dynamics*, *JHEP* **02** (2000) 020 [hep-th/9912072].

[8] H. Grosse and R. Wulkenhaar, *Renormalisation of ϕ^4-theory on non-commutative R^4 to all orders*, *Lett. Math. Phys.* **71** (2005) 13 [hep-th/0403232].

[9] H. Grosse and R. Wulkenhaar, *Renormalisation of ϕ^4 theory on noncommutative R^4 in the matrix base*, *Commun. Math. Phys.* **256** (2005) 305 [hep-th/0401128].

[10] H. Grosse, private communication.

[11] R. Gurau, J. Magnen, V. Rivasseau and A. Tanasa, *A translation-invariant renormalizable non-commutative scalar model*, arXiv:0802.0791.

[12] A. Matusis, L. Susskind and N. Toumbas, *The IR/UV connection in the non-commutative gauge theories*, *JHEP* **12** (2000) 002 [hep-th/0002075].

[13] M. Van Raamsdonk, *The meaning of infrared singularities in noncommutative gauge theories*, *JHEP* **11** (2001) 006 [hep-th/0110093].

[14] A. Armoni and E. Lopez, *UV/IR mixing via closed strings and tachyonic instabilities*, *Nucl. Phys.* **B 632** (2002) 240 [hep-th/0110113].

[15] Z. Guralnik, R.C. Helling, K. Landsteiner and E. Lopez, *Perturbative instabilities on the non-commutative torus, Morita duality and twisted boundary conditions*, *JHEP* **05** (2002) 025 [hep-th/0204037].

[16] P. Castorina and D. Zappala, *Nonuniform symmetry breaking in noncommutative $\lambda\ \phi^4$ theory*, *Phys. Rev.* **D 68** (2003) 065008 [hep-th/0303030].

[17] W. Bietenholz, F. Hofheinz and J. Nishimura, *Numerical results on the non-commutative $\lambda\phi^4$ model*, *Nucl. Phys.* **129** *(Proc. Suppl.)* (2004) 865 [hep-th/0309182].

[18] W. Bietenholz, J. Nishimura, Y. Susaki and J. Volkholz, *A non-perturbative study of* 4*D* U(1) *non-commutative gauge theory: the fate of one-loop instability*, *JHEP* **10** (2006) 042 [hep-th/0608072].

[19] M. Panero, *Quantum field theory in a non-commutative space: theoretical predictions and numerical results on the fuzzy sphere*, *SIGMA* **2** (2006) 081 [hep-th/0609205].

[20] M. Panero, *Numerical simulations of a non-commutative theory: the scalar model on the fuzzy sphere*, *JHEP* **05** (2007) 082 [hep-th/0608202].